SHM L.C. Hons 1980

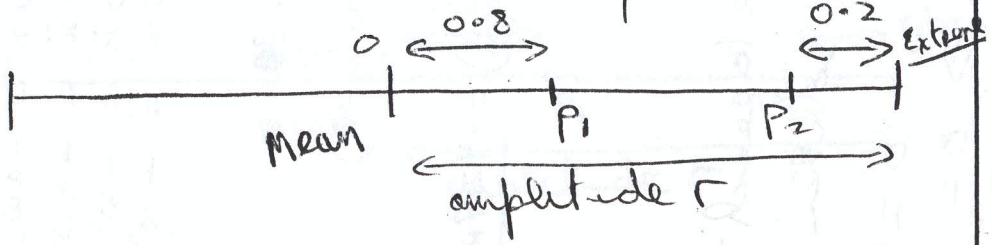
No. 13

$x = r\cos\omega t$.

Show Sim: $\frac{dx}{dt} = -\omega r \sin\omega t$

$\frac{d^2x}{dt^2} = -\omega^2 r\cos\omega t$

accel $= -\omega^2 x$ qed.



At P_1: $x = .8$m, $v_1 = 6\,ms^{-1}$

At P_2: $x = (r - 0.2)$, $a = 24$ in magnitude.

P_1: $v^2 = \omega^2(A^2 - x^2) \Rightarrow 6^2 = \omega^2(r^2 - (0.8)^2)$

$\Rightarrow 36 = \omega^2(r^2 - 0.64)$ (1)

P_2: $|accel| = \omega^2 x \Rightarrow 24 = \omega^2(r - 0.2)$ (2)

$\frac{(2)}{(1)} \Rightarrow \frac{24}{36} = \frac{\omega^2(r-0.2)}{\omega^2(r^2-0.64)}$

$\Rightarrow \boxed{\frac{2}{3} = \frac{r-0.2}{r^2-0.64}}$ qed

Next find r

$\Rightarrow 2(r^2 - 0.64) = 3(r - 0.2)$

$\Rightarrow 2r^2 - 1.28 = 3r - 0.6$

$\Rightarrow 2r^2 - 3r - 0.68 = 0$.

$\Rightarrow r = -0.2$ or $r = 1.7$.

$r = -0.2$ has no meaning here so pick $\boxed{r = 1.7\,m}$

Find T. (Find ω)

(2) $\Rightarrow 24 = \omega^2(1.7 - 0.2)$

$\Rightarrow 24 = \omega^2(1.5)$

$\Rightarrow 16 = \omega^2$

$\Rightarrow \boxed{4 = \omega}$

Find shortest time to travel from P_1 to P_2

Time to travel o to P_1:

$x = .8$ $\therefore x = A\sin\omega t$ (started at centre)

$\Rightarrow .8 = 1.7\sin 4t$

$\Rightarrow .4706 = \sin 4t$

$\Rightarrow .49 = 4t$

$\Rightarrow .1225 = t$

See Board for Cos alternative.

P_2: $1.5 = A\cos\omega t$

P_1: $.8 = A\cos\omega t$

Time to travel o to P_2

$x = 1.7 - .2 = 1.5$ $\therefore x = A\sin\omega t$.

$\Rightarrow 1.5 = 1.7\sin 4t$

$\Rightarrow .8824 = \sin 4t$

$\Rightarrow 1.0809 = 4t$

$\Rightarrow 0.2702 = t$

Time to travel P_1 to P_2 $= .2702 - .1225 = .1477$ sec.